5. Kolmanovskii, V. B. and Chernous'ko, F. L., Optimization of noise when observing a dynamic system. Izv. Akad. Nauk SSSR, Tekhnicheskaia Kibernetika, №2, 1972.
6. Fikhtengol'ts, G. M., Course on Differential and Integral Calculus, Vol. 3, Moscow, "Nauka", 1966.
7. Neustadt, L. W., Optimization, a moment problem and nonlinear programing. J. Soc. Industr. Appl. Math., A. Vol. 2, №1, 1964.
8. Kolmogorov, A. N. and Fomin, S. V., Elements of Function Theory and of Functional Analysis. Moscow, "Nauka", 1968.
9. Mishina, A. P. and Proskuriakov, I. V., Higher Algebra. Moscow, "Nauka", 1965.

Translated by N.H.C.

UDC 533.6

ON THE POSSIBILITY OF GASDYNAMIC EFFECTS AT THE CRITICAL POINT OF THE PHASE EQUILIBRIUM

PMM Vol. 37, №1, 1973, pp. 169-171
V. I. TSURKOV
(Moscow)
(Received September 28, 1971)

A possibility is indicated of appearance of density excursions in one-dimensional unsteady fluid flows near the critical point of the phase equilibrium, resulting from the singularities in the equation of state.

The present investigations are concerned with the question, whether the classical solutions of the problem and the initial conditions for the one-dimensional unsteady gasdynamic equations can become infinite in the nonisoentropic case. Here we have to consider a system of three quasilinear hyperbolic equations which, as we know [1, 2], usually have unbounded solutions. On the other hand, the system of gasdynamic equations has a number of specific properties. Of those the most important is the presence of a single invariant, i.e. of a function which remains bounded [1]. Another important property consists of the fact that the generalized Riemann invariants satisfy multi-dimensional integral equations of Volterra type, in which the cone of integration is represented by the domain of definition of the hyperbolic equations and the boundedness of the solution follows from the fine properties of the integrability of the kernel. In the terms of the gasdynamic equations the latter lead to restrictions imposed on the equations of state. The properties themselves follow from the boundedness of the variation of entropy along the sonic characteristics and from the weak linearity (tangency) of the entropic characteristics [3].

The conditions which must be imposed on the equations of state in order to secure the boundedness, are expressed by the following inequalities [3]

$$0 < c^\circ \leqslant c_V = T\left(\frac{\partial S}{\partial T}\right)_V, \qquad \left|\left(\frac{\partial^2 p}{\partial \rho \partial S}\right)\left(\frac{\partial p}{\partial \rho}\right)_S^{-1}\right| \leqslant K^\circ \tag{1}$$

Here ρ is the density, S is entropy per unit mass, V is the specific volume, $p = p(\rho, S)$ is the pressure, T is the temperature and c_V is the heat capacity of a unit mass.

We consider the case when the second of the above restrictions does not hold [3]. This occurs at the critical point of the phase equilibrium in accordance with the phenomenological theory [4] based on the fact that heat capacity has a logarithmic singularity. A possibility is indicated of appearance of gasdynamic effects, consisting of localized unbounded density excursions and associated with the properties arizing from the unboundedness of the solution of the gasdynamic equations obtained in [3, 5].

We consider the Cauchy problem for the gasdynamic equations in Lagrangian coordinates with the dissipative terms absent

$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial q} = 0, \qquad \frac{\partial u}{\partial t} + \frac{\partial p}{\partial q} = 0, \qquad \frac{\partial s}{\partial t} = 0$$

$$u(0, q) = 0,\ v(0, q) = 0,\ \theta_i(0, q) = \theta_i^\circ(q),\ q \in [a, b],$$
$$s = S - S_*,\ v = V - V_*,\ \theta = T - T_*$$

where S_*, T_* and V_* denote the values which the thermodynamic functions assume at the critical point, t is time, q is the Lagrangian mass coordinate and u is velocity.

It is assumed that $\theta_i(q)$ is a sequence of symmetric nonnegative smooth functions possessing a unique local minimum at the point $q = 0$, the minimum tending to zero along the functional sequence. In other words, at the initial instant we have a homogeneous phase with critical density, and the state at the point $q = 0$ is nearly critical.

The pressure near the critical point is composed of two parts [4], the regular part p_1 and the irregular part p_2. The regular part is represented by a series in the powers of θ and v and, within the accuracy of up to the higher order terms, it has the following form [6]

$$p_1 = -A\theta v - Bv^3 / 3 + f(\theta) \quad (A = \text{const},\ B = \text{const})$$

where $f(\theta)$ is an undefined regular function. In accordance with [4] the principal term of the expansion for $f(\theta)$ near the critical point is equal to $E\theta$, where $E = \text{const} > 0$. The irregular part is equal to $(\partial F_2 / \partial v)_\theta$, where F_2 represents an irregular supplement to the free energy [4]

$$F_2 = \alpha_1 \theta^2 \ln[(\theta + \beta v^2)^2 + \gamma^2 v^4] + \theta^2 h(\theta / v^2)$$

Here h is a bounded function (of no importance in what follows), α_1, β and $\gamma = \text{const}$, and $\alpha_1 < 0$.

The irregular supplement to the heat capacity is [4]

$$c_{V_2} = 2\alpha_1 \ln[(\theta + \beta v^2)^2 + \gamma^2 v^4] + h_1(\theta / v^2) \tag{2}$$

where h_1 is another bounded function also unimportant in the following.

It can be shown that under these conditions the adiabatic speed of sound $a = (\partial p / \partial \rho)_S^{1/2}$ tends to zero on approaching the critical point. We also find that as $(\partial^2 p / \partial \rho \partial S)(\partial p / \partial \rho)_S^{-1} = (\partial \ln a^2 / \partial S)$, the second restriction of (1) does not hold.

Let us now write an expression for the pressure near the critical point as the function of the density and specific entropy during, at least, the initial instant, i.e. when $v = 0$, so that the results of [3, 5] can be subsequently applied.

By (2) we have the following relation for the derivative $(\partial S / \partial \theta)_v$ near the critical point

$$(\partial S / \partial \theta)_v = C_{V_1} / T + (2\alpha_1 / T) \ln[(\theta - \beta v^2)^2 + \gamma^2 v^4] + [h_1(\theta / v^2)] / T$$

where c_{V_1} denotes the regular part of the heat capacity.

In the exact theory [4] the part of the coefficients accompanying the logarithmic terms in the irregular parts of the free energy and heat capacity is played by the undetermined regular functions of θ and v which are not zero when $\theta = v = 0$. It can therefore be assumed that the logarithmic part of the derivative $(\partial S / \partial\theta)_v$ near the critical point, which will be important in what follows, has the form

$$(\partial S / \partial\theta)_{v\log} = \alpha_2 V^{\gamma_1} \ln[(\theta + \beta v^2)^2 + \gamma^2 v^4], \qquad (\alpha_2,\ \gamma_1 = \text{const},\ \alpha_2 < 0,\ \gamma_1 > 0) \quad (3)$$

Let us now replace (3) by

$$(\partial s / \partial\theta)_v = \alpha_3 V^{\gamma_1} (\theta + \beta v^2)^{-\varepsilon} \quad (\alpha_3 = \text{const} > 0) \qquad (4)$$

where ε is a small positive number. This means that we have replaced the logarithmic growth of heat capacity by one increasing as a small power, which simplifies the computations considerably without affecting the final result. Moreover we achieve further simplification by neglecting the term $\gamma^2 v^4$, as its effect in the initial form of the Cauchy problem at the first instant, may be assumed insignificant.

Integrating (4) we obtain, with the accuracy of up to an arbitrary function v, the following principal contribution to the entropy

$$s = \alpha_3 V^{\gamma_1} (\theta + \beta v^2)^{(1-\varepsilon)} / (1 - \varepsilon)$$

from which we have

$$\theta = (1 - \varepsilon)^{\omega} \rho^{\gamma_1 \omega} s^{\omega} \alpha_3^{-\omega} - \beta v^2 \qquad (\omega = 1 / (1 - \varepsilon))$$

The corresponding principal contribution to the irregular part of the pressure has the form

$$p_2 = \frac{\alpha_3 \gamma_1 V^{\gamma_1 - 1} (\theta + \beta v^2)^{(2-\varepsilon)}}{(1 - \varepsilon)(2 - \varepsilon)} + \frac{\alpha_3 V^{\gamma_1} (\theta + \beta v^2)^{(1-\varepsilon)}\, 2\beta v}{(1 - \varepsilon)}$$

Since $v = 0$ and $p_2 \sim \theta^{(2-\varepsilon)}$ we find, when investigating the initial Cauchy problem, that the principal contribution is made by the term $E\theta$ of the regular part of the pressure. When $t = 0$, this contribution is

$$p_1 = E (1 - \varepsilon)^{\omega} \rho^{\gamma_1 \omega} s^{\omega} \alpha_3^{-\omega} \qquad (5)$$

Equation (5) is of the type $p = A^2{}_0 \rho^{\gamma_0} S^{2\alpha} / \gamma_0$, where γ_0 is replaced by $\gamma_1 \omega$ and $2\alpha = \omega$. When the Cauchy problem with the initial entropy $S^0(q) = S^0 q^{\beta} + S^i_{\min}$ and density $\rho^0(q) = \rho^0(\rho^0, S^0, \beta, S^i_{\min} = \text{const} > 0)$ was considered, the solution obtained in [3, 5] for this class of equations of state behaved as follows. The pressure gradients by virtue of the entropy gradients, force the gas to flow into the point $q = 0$. If $\alpha\beta \in [0.5, 1]$ and the maximum density at the point $q = 0$ in the corresponding solution tends to infinity as $S^i_{\min} \to 0$. When $\alpha\beta = 1$, the time in which the maximum is reached, tends to a finite value (the "break-through"-type unboundedness), when $\alpha\beta \in (0.5, \alpha\beta^*(\gamma_0))$, the time tends to zero (the "excursion"-type unboundedness [5]).

Considering the sequence of temperatures $\theta^0(q) = \theta^0 q^{\mu} + \theta^i_{\min}$ and $\theta^i_{\min} \to 0$, where θ^0, $\mu = \text{const} > 0$ near the critical point we find, that as $S^0(q) \sim q^{\mu\omega}$, the condition $0.5 < \alpha\beta < 1$ yields $1 < \mu < 2$. Naturally the density does not become infinite because, e, g, the expression for p_1 contains another term which gives rise to counter pressure with increasing density. All the same, if formally $B \to 0$, then the maximum density tends to infinity, i.e. at some small values of B one can obtain a strong, localized density peak near the critical point of the phase equilibrium.

Note. It was communicated to the author by the referee that density excursions are indeed observed in practice in the energy generators, in the flows near the critical point of the phase equilibrium.

In conclusion the author thanks L. A. Galin, S. I. Anisimov and M. Ia. Azbel' for assessment of this paper.

BIBLIOGRAPHY

1. Molchanov, A. M., Boundedness of variation of continuous solutions of the system of hydrodynamic equations. Dokl. Akad. Nauk SSSR, Vol. 129, №6, 1959.
2. Rozhdestvenskii, B. L. and Ianenko, N. N., System of Quasilinear Equations. M., "Nauka", 1968.
3. Tsurkov, V. I., Gasdynamic effects related to the boundedness of solutions of quasilinear systems. J. USSR Comput. Math. and Math. Phys., Vol. 11, №2, Pergamon Press, 1971.
4. Azbel', M. Ia., Voronel', A. V. anf Giterman, M. Sh., On the theory of critical point. ZhEFT, Vol. 46, №2, 1964.
5. Tsurkov, V. I., On a selfsimilar solution of the gasdynamic equations. J. USSR Comput. Math. and Math. Phys., Vol. 11, №4, Pergamon Press, 1971.
6. Landau, L. D. and Lifshits, E. M., Theoretical Physics, Vol. 5, Statistical Physics (English translation), Pergamon Press, 1968.

Translated by L. K.

UDC 534.222.2

EXPLOSIONS IN DETONATING MEDIA OF VARIABLE INITIAL DENSITY

PMM Vol. 37, №1, 1973, pp. 172-177
N. S. MEL'NIKOVA
(Moscow)
(Received May 6, 1971)

We consider a non-self-similar problem of point explosion in a detonating gas, in a medium of variable initial density. Analytic expressions are obtained showing the dependence of the pressure, density and gas velocity on the distance from the origin of explosion and the radius of detonation wave, the latter obtained by solving a differential equation. Computations are performed for the cases of spherical and cylindrical symmetry for various values of the adiabatic exponent, and the variation of initial density exponent.

Let us consider a perfect gas which is inviscid and non-heat-conducting. Suppose that an instantaneous explosion of finite energy E_0 occurs at the instant $t = 0$ in an unbounded medium at rest $(v_1 = 0)$ at a point, or along a plane, or along a staight line [1]. The explosion generates a strong shock wave which propagates through the gas and heats it up to the state at which rapid combustion becomes possible. Assuming that the energy E_0 is large and much larger than the amount of energy Q_1 released during the gas combustion, we can infer that the gas burns in the direct vicinity of the shock-wave front. In this case we can consider the shock wave and the chemical reaction zone together, as a single surface of a strong explosion with release of heat, i. e. treat it as a detonation-wave front.